Symplectic Integrators and N-body

10-6-2006

Opening Discussion

- Do you have any questions about the quiz?
- What did we talk about last class?

Symplectic Integrators

- You have seen that ode45 fails to do a good job of conserving energy in the systems we have given it. We could try to increase the accuracy, but that's just a stop-gap. We really need a different type of integrator.
- To understand symplectic integrators we should talk briefly about Hamiltonian systems. They are defined by a value H(p,q) which is basically the total energy of the system in terms of momenta (p) and positions (q) of the bodies.
- For Hook's law $H=0.5*p^2/m+0.5*kq^2$. This is just kinetic plus potential energy. The time derivatives of p and q are given by the partial derivatives of H.

The Leapfrog

- We can build a symplectic integrator by breaking the problem into pieces that we can solve exactly, then alternating between solving those pieces (this is a simplified view). We will use a T+V style also known as a kick-step method.
- Given the current position we calculate change in momentum and apply that. Then we take a step using the new momentum. Simply repeat this for the integration.
- This will perfectly integrate some Hamiltonian system that is similar to the one we are really interested in.

Project

- I want the two of you to pick a Hamiltonian N-body system to integrate and test at least three different integration schemes on it.
  - Ode45
  - Euler
  - Leapfrog
- Your write-up should include analysis of how well they conserve the Hamiltonian and how fast they are when you want the Hamiltonian conserved to a certain level.
- You can also try seeing what you can do in Matlab to make it more efficient for large N.

Closing Comments

- No class next week. Write me e-mails with questions.
- The test is Wednesday after I get back and the project is due that Friday.