Approximation Algorithms

4-25-2006

Opening Discussion

- What did we talk about last class?

Randomized MAX-3-CNF

- The MAX-3-CNF is an optimization form of the NP 3-CNF. Instead of asking whether there is a solution fro which all terms are true, we ask what set of inputs gives us the most true terms.
- We can get an 8/7-approximation algorithm simply by selecting the values for the inputs at random. This works because each term has three variables (or their negations) in it so each one will have a ½ chance of being true. The three together have a 1/8 chance they are all false or 7/8 chance at least one is true.
- If we treat true as 1, our expected value for each term is 7/8 for each one that can be satisfied.

Weighted Vertex Covering

- The weighted vertex covering adds a weight to each of the vertices in an undirected graph and we have to find a covering set with a minimal sum for the weights.
- We can't approximate this the way we could with a standard minimal vertex covering and hope to get a good approximation.
- We can approximate this with linear programming. Ideally our variables are x(v) with a value of 0 or 1. However, that doesn't help us since integer linear programming isn't tractable. Instead, we allow x(v) to vary between 0 and 1.

LP Weighted Vertex Covering

- This gives us the following linear programming problem.
  - Minimize sum(w(v)*x(v))
  - x(u)+x(v)>=1 for each (u,v) in E
  - x(v)<=1
  - x(v)>=0
- The solution to this will be a 2-approximation to the full problem.

Subset-Sum Problem

- Here is our last approximation algorithm. Given a set of non-negative numbers, S, we want to find the subset that is as close to a value, t, without exceeding it.
- A general solution can be made by continually building a list that contains the sums of all subsets of S iteratively putting in more of the elements of S. We can prune the list by taking out any elements greater than t. This is exponential.
- We can get an approximation by keeping the list short through trimming. This process removes any element y if there is an element z such that $y/(1+d) \le z \le y$.

Number Theory Stuff

- Cryptography – factoring is hard.
- Modular arithmetic

Reminders

- Remember to turn in test 7 next class.